QP CODE: 24803771

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Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, JUNE 2024

Second Semester

INTEGRATED MSC BASIC SCIENCE-PHYSICS

COMPLEMENTARY - IPH2CM05 - MATHEMATICS - II INTEGRAL CALCULUS AND FOURIER SERIES

2021 Admission Onwards

644262E1

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Compute $\int_0^b x\, dx$.
2. Find $\int x^2 \sin(x^3) dx$.
3. Find $\int_{\pi/4}^{\pi/2} \cot\theta \cos^2\theta\, d\theta$.
4. Define the arc length function for $y = f(x)$.
5. Define the area of the surface generated by revolving the curve $x = g(y)$ about the y-axis.
6. Evaluate $\int_0^1 \int_0^2 x(x+y) dy\, dx$.
7. Write three properties of triple integrals.
8. Find the fundamental period of the functions (a) $\cos(\pi x)$ (b)$\sin(x)$.
9. Write Fourier series expansion of $f(x)$ of period $p = 2\pi$ and give the Euler formula for the coefficients.
10. Find the Taylor polynomial of order 2 generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Evaluate the integral
    (a) $\int_{-3}^{4}(5 - \frac{x}{2})dx$
    (b) $\int_0^5 x^{3/2} dx$

12. Find $\int x \sin 2x^2 dx$.

13. The solid lies between the planes perpendicular to the y-axis at $y = 0$ and $y = 2$. The cross sections perpendicular to the y-axis are circular disks with diameters running from y-axis to the parabola $x = \sqrt{5}\, y^2$.. Find the volume of the solid.

14. The region in the first quadrant enclosed by the parabola $y = x^2$, the y-axis and the line $y = 1$ is revolved about the line $x = \frac{3}{2}$ to generate a solid. Find the volume of the solid.

15. Find the volume of the prism whose base is the traingle bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$.

16. Sketch the region and evaluate the area of $-1 \leq x \leq 0, -2x \leq y \leq 1 - x$.

17. Find the Fourier coefficients of the periodic function $f(x)$ with period 2π,
$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$ & $f(x + 2\pi) = f(x)$.

18. Find the fourier sine series expansion of the function $f(x) = \pi - x,\ 0 < x < \pi$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) Find $\int_0^1 (4y - y^2 + 4y^3 + 1)^{-2/3} (12y^2 - 2y + 4)\, dy$.
(b) Find $\int_0^{\sqrt[3]{\pi^2}} \sqrt{\theta} \cos^2 (\theta^{3/2})\, d\theta$.

20. (a) Find the length of $y^2 = x^3$ extending from origin to $(1, 1)$.
(b) Find the length of $y = \log \sec x$ between the points given by $x = 0$ and $x = \pi/3$.

21. Find the average value of $f(x, y, z) = xyz$ over the cube bounded by the co-ordinate planes and the planes $x = 2, y = 2, z = 2$ in the first octant.

22. a) State whether the function $f(x) = \begin{cases} k & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$ is even or odd. Find its Fourier series and also deduce the value of the seires $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

b) State whether the function $f(x) = \begin{cases} -2x & \text{if } --\pi < x < 0 \\ 2x & \text{if } 0 < x < \pi \end{cases}$ is even or odd. Find its Fourier series and also deduce the value of the seires $\frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \ldots$

(2×5=10 weightage)